STRUCTURAL ANALYSIS I

UNIT-1

DEFLECTION OF DETERMINATE STRUCTURES

CONTENTS

TECHNICAL TERMS

Law of conservation of energy:

Energy neither be created nor be destroyed.

Virtual work:

The term virtual work means the work done by real force acting through a virtual displacement or virtual force acting through a real displacement.

Determinate structure:

The structure in which the number of unknown reactions are equal to the number of available static equilibrium equations.

Continuous Beam:

A Beam which is supported on more than two supports is called a continuous Beam.

Propped Cantilever Beam:

A propped cantilever beam is a beam in which one end is fixed and is vertically supported with a prop at the free end or at any intermediate span.

Williot diagram:

It is a graphical method to obtain an approximate value for displacement of a structure which submitted to a certain load.

Truss:

A truss is a structure comprising one or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes.

Frames:

Frames usually consist of post and beam or slab and wall systems, but can also be configured using combinations of structural members.

UNIT-1

DEFLECTION OF DETERMINATE STRUCTURES

1.1 PRINCIPLES OF VIRTUAL WORK FOR DEFLECTIONS

1.1.1 Theorem of minimum Potential Energy

Potential energy is the capacity to do work due to the position of body. A body of weight „W" held at a height „h" possess energy „W_h". Theorem of minimum potential energy states that " Of all the displacements which satisfy the boundary conditions of a structural system, those corresponding to stable equilibrium configuration make the total potential energy a relative minimum".

1.1.2 Law of Conservation of Energy

From physics this law is stated as "Energy is neither created nor destroyed". For the purpose of structural analysis, the law can be stated as "If a structure and external loads acting on it are isolated, such that it neither receive nor give out energy, then the total energy of the system remain constant". With reference to figure 2, internal energy is expressed as in equation (9). External work done W_e = -0.5 P dL. From law of conservation of energy U_i+W_e =0. From this it is clear that internal energy is equal to external work done.

1.1.3 Virtual work for deflections:

This method is sometimes referred to as the unit-load method. The term virtual work means the work done by real force acting through a virtual displacement or virtual force acting through a real displacement. The virtual work is not a real quantity but an imaginary one. The principle of virtual work is based on the conservation of energy for a structure, which implies the work done on a structure by external loads is equal to the internal energy stored in the structure.(U_e=U_i) .

Take a deformable structure of any shape or size and apply a series of external loads {P} to it. It will cause internal forces {F} at points throughout the structure. It is necessary that the internal and external loads to be related by equations of equilibrium. As a consequence of above loadings, external displacements Δ will occur at the P loads and internal displacements{δ} will occur at each point of internal forces{F}.

In general, the principle of virtual work and energy states that

$$\underbrace{\Sigma P \Delta}_{\text{Work of External loads}} = \underbrace{\Sigma F \delta}_{\text{Work of internal forces}}$$

1.2 DEFLECTIONS OF PIN - JOINTED PLANE FRAMES

Sign convention:

Assume that tensile forces are positive (+) and compressive forces are negative (-).

PROCEDURE FOR ANALYSIS:

Step: 1

Calculation of virtual forces (K)

Remove all the real load from the truss. Place a unit load on the truss at the joint and in the direction of the desired displacement. Use the methods of joints or the method of sections and calculate the internal forces k in each member of the truss.

Step: 2

Calculation of Real forces (F)

These forces are caused only by the real loads acting on the truss. Use the methods of joints or the method of sections to determine the forces F in each member.

Step: 3

Virtual work equation

Apply the equation of virtual work, to determine the desired displacement.

$$1.\Delta = \Sigma \frac{kFL}{AE}$$

Example 1.1 Determine the vertical displacement of joint C of the steel truss shown in figure. The cross sectional area of each member is A=400mm^2 and E=2x10^5 N/ mm^2.

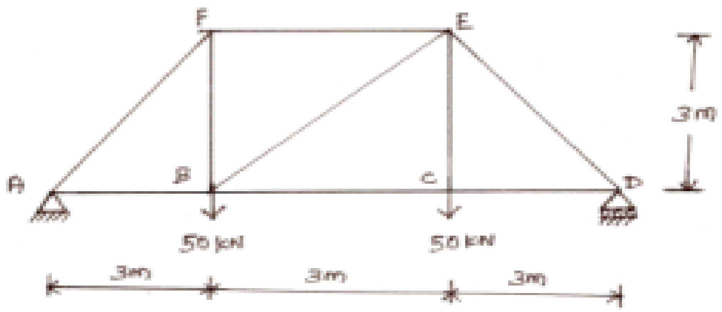


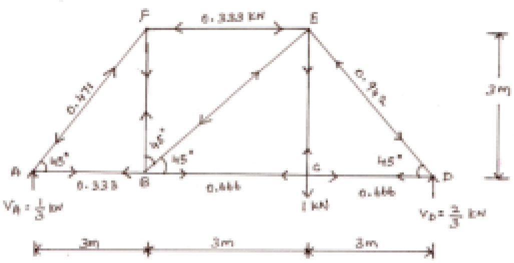
Fig.1.1

Solution:

We know that

$$1.\Delta = \Sigma \frac{kFL}{AE}$$

Virtual forces (K): Remove all the external loads and apply a unit vertical force at joint C of the truss. Analyse the truss using the method of joints.



Take moments about D,

$V_A \times 9 - 1 \times 3 = 0$

$V_A \times 9 = 1 \times 3$

$V_A = 1/3$ kN

$V_D =$ Total load-V_A

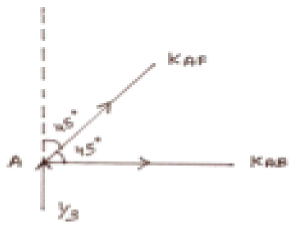
$= 1 - 1/3 = 2/3$ kN

Joint A:

Initially assume all forces to be tensile.

$\Sigma V=0$ gives

$k_{AF} \cos 45^0 + 1/3 = 0$



$k_{AF} \cos 45^0 = -1/3$

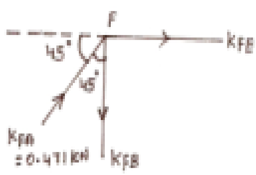
$k_{AF} = - 1/(3 \times \cos 45^0) = -0.471$ kN

$k_{AF} = 0.471$ kN(compression)

$\Sigma H=0$ gives $k_{AF} \cos 45^0 + k_{AB} = 0$

$(-0.471) \cos 45^0 + k_{AB} = 0$

$k_{AB} = 0.333$ kN(Tensile)



Joint F:

$\Sigma H=0$ gives $k_{FA} \cos 45^0 + k_{FE} = 0$

$k_{FE} = - k_{FA} \cos 45^0$

$= - 0.471 \times \cos 45^0$

$= - 0.333$ kN (comp)

$\Sigma V=0$ gives

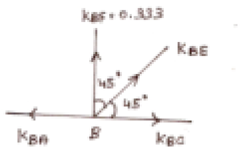
$k_{FA} \cos 45^0 - k_{FB} = 0$

$k_{FA} \cos 45^0 = k_{FB}$

$k_{FB} = 0.471 \text{ x } \cos 45^0$

$k_{FB} = 0.333$ kN(Tensile)

Joint B:



ΣV=0 gives

$K_{BE} \cos 45^0 + k_{BF} \ \ =0$

$K_{BE}= - k_{BF}/ \cos 45^0 = - 0.333/ \cos 45^0 = - 0.471$

$K_{BE} = 0.471$ kN (comp)

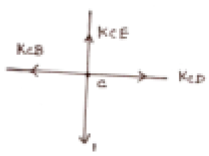
ΣH=0 gives $k_{BC} - k_{BA+} k_{BE} \cos 45^0 =0$

$k_{BC} = k_{BA} - k_{BE} \cos 45^0$

$= 0.333- (-0.471) \cos 45^0$

$k_{BC} = 0.666$ kN (Tensile)

Joint C:



ΣV=0 gives K_{CE} **– 1= 0**

K_{CE} = 1kN (Tensile)

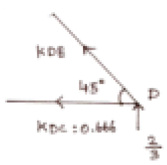
ΣH=0 gives

$k_{CD} - k_{CB} =0$

$k_{CD} = k_{CB} = 0.666$

$k_{CD} =$ 0.666kN(Tensile)

Joint D:



ΣH=0 gives

$K_{DE} \cos 45^0 + k_{DC} = 0$

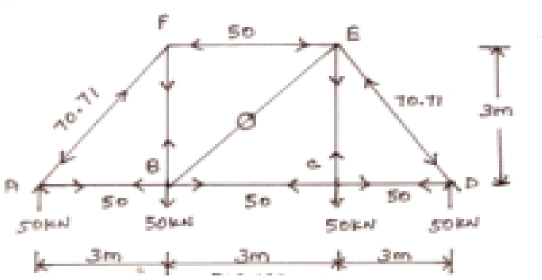
$K_{DE} = - k_{DC} / \cos 45^0$

$= - 0.666 / \cos 45^0 = - 0.942$

$K_{DE} = 0.942$kN (comp)

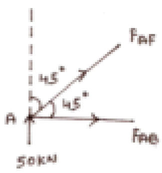
Real forces, F:

The real forces in the members due to the given system of external loads are calculated using method of joints.



By symmetry $R_A = R_D =$ Total load/2 =50 kN

Joint A:



ΣV=0 gives $F_{AF} \cos 45^0 + 50 = 0$

$F_{AF} \cos 45^0 = -50$

$F_{AF} = - 50 / \cos 45^0 = -70.71$ kN

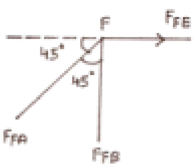
$F_{AF} = 70.71$ kN (compression)

ΣH=0 gives $F_{AF} \cos 45^0 + F_{AB} = 0$

$(-70.71) \cos 45^0 + F_{AB} = 0$

$F_{AB} = 70.71 \times \cos 45^0 = 50$ kN (Tensile)

Joint F:



$\Sigma H=0$ gives $F_{FA}\cos 45^0 - F_{FB} = 0$

$-F_{FB} = -F_{FA}\cos 45^0$

$= 70.71 \times \cos 45^0$

$F_{FB} = 50$ kN (Tensile)

$\Sigma V=0$ gives

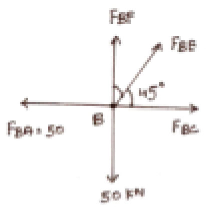
$F_{FA}\cos 45^0 + F_{FE} = 0$

$F_{FE} = -F_{FA}\cos 45^0$

$F_{FE} = -70.71 \times \cos 45^0$

$F_{FE} = 50$ kN (compression)

Joint B:



$\Sigma V=0$ gives

$F_{BE}\cos 45^0 + F_{BF} - 50 = 0$

$F_{BE}\cos 45^0 = -50 + 50$

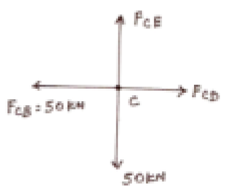
$F_{BE} = 0$

$\Sigma H=0$ gives $F_{BC} - F_{BA} + F_{BE}\cos 45^0 = 0$

$F_{BC} - 50 + 0 = 0$

$F_{BC} = 50$ kN (Tensile)

Joint C:



$\Sigma V=0$ gives

F_{CE} **– 50= 0**

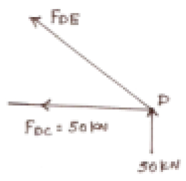
$F_{CE} = 50$kN (Tensile)

$\Sigma H=0$ gives $F_{CD} - F_{CB} = 0$

$F_{CD} = F_{CB} = 50$

$F_{CD} = 50$ kN (Tensile)

Joint D:



ΣH=0 gives

$F_{DE} \cos 45^0 + F_{DC} = 0$

$F_{DE} = - F_{DC} / \cos 45^0$

$= - 50/ \cos 45^0 = - 70.71$

$F_{DE} = 70.71$ kN (comp)

Virtual work equation:

$$1.\Delta = \Sigma \frac{kFL}{AE}$$

$AF=BE=DE=\sqrt{3^2+3^2}$

=4.243m

S.No.	Member	K	F (kN)	L (m)	kFL (kNm)
1	AF	-0.471	-70.71	4.243	141.311
2	FE	-0.333	-50.00	3.00	49.950
3	ED	-0.942	-70.71	4.243	282.621
4	DC	0.666	50.00	3.00	99.900
5	CB	0.666	50.00	3.00	99.900
6	BA	0.333	50.00	3.00	49.950
7	FB	0.333	50.00	3.00	49.950
8	BE	-0.471	0	4.243	0
9	EC	1.000	50.00	3.00	150.00
				ΣkFL=	923.582

$\Sigma kFL = 923.582 \times (1000)^2$ Nmm

$$\Delta c = \Sigma \frac{kFL}{AE}$$

$$= \frac{923.582 \times (1000)^2}{400 \times 2 \times 10^5}$$

$$= 11.54mm$$

Vertical displacement of joint C = 11.54mm

1.3 DEFLECTION OF BEAMS AND RIGID PLANE FRAMES

DEFLECTION OF BEAMS (Using the method of virtual work)

The method of virtual work may be formulated for the beam deflections by considering the beam as shown below.

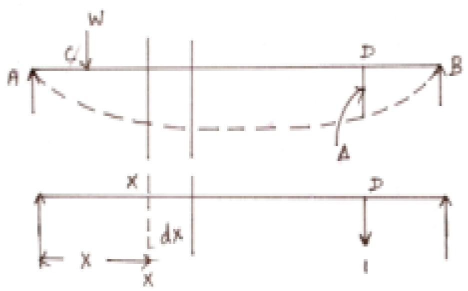


Fig.1.2

Here the vertical displacement Δ of point D is to be determined.To compute Δ a virtual unit load acting in the direction of Δ (Vertical) is placed on the beam at D, and the internal virtual moment *m* is determined by the method of sections at an atributary location x from the left support. When the real loads act on the beam, the point D is displacement downward by Δ. Then the element *dx* rotates by *d*θ[(=M/EI) *dx*].Here M is the internal moment at *x* caused by the real loads. Consequently, the external work done by the unit load is 1.Δ, and the internal virtual work done by the moment *m* is *m*. *d*θ= *m*(M/EI) *dx*. Summing the effects on all elements *dx* along beam requires an integration and hence

$$1.\Delta = \int_0^l m \frac{M}{EI} dx$$

1 = External virtual unit load acting on the beam in the desired direction of Δ

m = internal virtue moment in the beam

Δ = external displacement of the point caused by the real loads acting on the beam

M = internal moment in the beam expressed as a function of *x* and caused by a real loads

E = modulus of elasticity of the material

I = moment of inertia of cross-sectional area about the neutral axis

The slope dy/ dx or θ at a point on the beam‟s elastic curve can be determined by applying a unit couple or moment at the point, and the corresponding internal moments m_θ have to be determined. The work of the unit couple is 1. θ.

Then

$$1.\theta = \int_0^l m_\theta \frac{M}{EI} dx$$

Example 1.2 Using the method of virtual work, determine the vertical displacement of point B of the beam shown in figure. Take $E=2x10^5$ N/ mm^2, $I=825x10^7$mm^4.

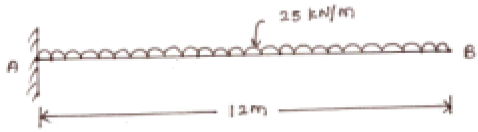


Fig. 1.3

Solution:

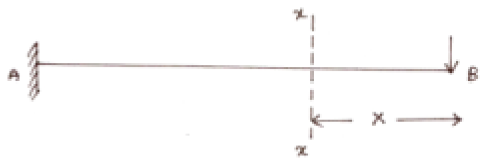
We know

$$1.\Delta = \int_0^l m \frac{M}{EI} dx$$

Virtual moment, m:

Remove the external load. Apply a unit vertical load at B. Consider a section *xx* at a distance *x* from B.

$m = -1.x$ (Hogging moment)

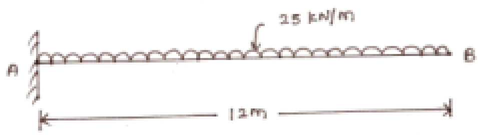


Real moment, M:

Using the same coo ordinate, the internal moment M is formulated as,

$M = - 25.x.x/2$

$M = - 12.5x^2$



Virtual work equation:

$$1.(\Delta_B)_V = \int_0^l m \frac{M}{EI} dx$$

$$= \int_0^{12} \frac{(-1.x)(-12.5x^2)}{EI}\, dx$$

=64800/EI

=0.0393m (or) 39.3m

Hence the vertical displacement of point B= 39.3mm.

1.4 WILLIOT DIAGRAM

It is a graphical method to obtain an approximate value for displacement of a structure which submitted to a certain load.

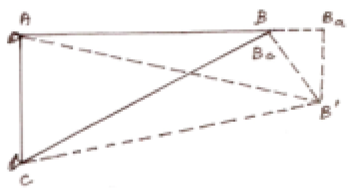


Fig. 1.4 Displacement diagram

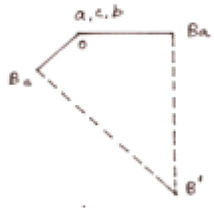


Fig. 1.5 Williots diagram

The method consists of, from a graph representation of a structural system, representing the structure's fixed vertices as a single, fixed starting point and from there sequentially adding the neighbouring vertices' relative displacements due to strain.

ASSUMPTIONS MADE IN THE UNIT LOAD METHOD

1) The external and internal forces are in equilibrium.

2) The supports are rigid and no moment is possible.

3) The material is strained well within the elastic limit.

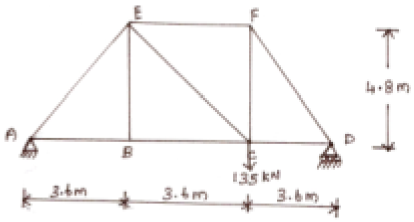
QUESTION BANK

PART-A

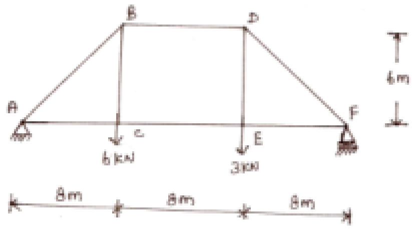
1. Define Virtual Work.
2. Differentiate between external and internal virtual work.
3. State principle of virtual displacement and principle of virtual forces.
4. Derive an expression of calculating deflections of structure using unit load method.
5. Calculate deflections of a statically determinate structure using unit load method.
6. State unit displacement method.
7. Calculate stiffness coefficients using unit-displacement method.
8. What is williot diagram?

PART-B

1. Using the method of virtual work, find the vertical deflection component of point E of the truss shown in figure. Cross-sectional areas of members are: AE and FD = 250mm^2 EF and EC = 1875mm^2; AB, BC, CD, EB and FC = 1250mm^2; E=200kN/mm^2.



2. For the truss shown in figure calculate the change in length of diagonal DC due to the applied loading. The area of upper and lower chords = 400mm^2, web members = 300mm^2 and E=200*10^3 N/mm^2 use the method of virtual work.



3. Using the method of virtual work, find the rotation at A of the beam shown in figure. E=200kN/mm^2; I= 1.66*10^{8}mm^4.

4. Determine the horizontal displacement at support D of the frame shown in figure. Relative I values are indicated along the members. E=200*106kN/m^2 and I=300*$10^{-6}$$m^4$. Use the principle of virtual work.