STRUCTURAL ANALYSIS I

UNIT IV

SLOPE DEFLECTION METHOD

CONTENTS

TECHNICAL TERMS

Slope deflection equations:

Slope deflection equations express the final end moments of each member in a frame

Stiffness:

Moment required to rotate one end by unit rotation, when rotation is permitted at the end.

Frames:

Frames usually consist of post and beam or slab and wall systems, but can also be configured using combinations of structural members.

Continuous Beam:

A Beam which is supported on more than two supports is called a Continuous Beam.

Relative stiffness:

When structural elements are linked so that their displacements are equal, they resist loads in proportion to their stiffness.

UNIT IV

SLOPE DEFLECTION METHOD

4.1 Introduction:

This is an algebraic method. Any indeterminate frame is made up of members in flexure. Solving the frame is the process of finding primarily the bending moments at the ends of each and every member. The moment M_A in a typical member AB is made up of 4 parts.

(a)M_{FAB} and M_{FBA}, the fixed end moments at the ends A and B due to the transverse loading on the member, when A and B are restrained from rotation or vertical displacement as in fig 4.1(a).

(b)Moments due to the rotation θ_A of A only.

(c) Moments due to the rotation θ_B of B only.

(d) Moments due to the deflections Δ_A and Δ_B at B.

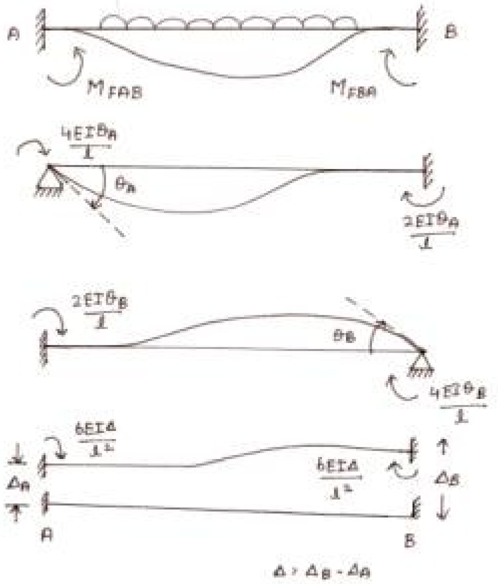


Fig.4.1 (*a*) to (*d*)

4.1.1 SIGN CONVENTION

For moments and rotations clockwise is positive (+).

For differential sinking, right upward movement is positive (+). For support moments, clockwise is positive (+).

4.1.2 SLOPE DEFLECTION EQUATIONS

Slope deflection equations express the final end moments of each member in a frame in terms of,

(i) Fixed end moments due to external loads.

(ii) Moments due to rotation at A (to the final value of rotation θ_A).

(iii) Moments due to rotation at B.

(iv) Moments due to differential transverse displacement of B above A.

The slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{4EI\theta}{l} + \frac{2EI\theta}{l} + \frac{6EI\Delta}{l^2}$$

If I/ l is taken as k (Stiffness)

$$M_{AB} = M_{FAB} + 2Ek\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$M_{AB} = M_{FBA} + 2Ek\left[\theta_A + 2\theta_B + \frac{3\Delta}{l}\right]$$

Employing slope deflections equations

To solve any indeterminate flexural frame by slope deflection method the following steps are adopted.

1) For all members write down the fixed end moments due to the given loading.

2) For each member write down the slope deflection equation in terms of the joint rotations and displacements.

3) At every joint several members join. Each member has moment acting on it. Write down joint equilibrium for moments is $M_{CA}+M_{CB}+M_{CD}=0$.

4) Solve the equilibrium equation to get the unknown displacement.

5) Use these values in slope deflection equations in step2 to obtain final joint moments.

6) Combine the joint moments of each member with the free bending moments to get the final bending moment in each member.

4.1.3 Application of Slope-Deflection Equations to Statically Indeterminate Beams

The procedure is the same whether it is applied to beams or frames. It may be summarized as follows:

1. Identify all kinematic degrees of freedom for the given problem. This can be done by drawing the deflection shape of the structure. All degrees of freedom are treated as unknowns in slope-deflection method.

2. Determine the fixed end moments at each end of the span to applied load. The table given at the end of this lesson may be used for this purpose.

3. Express all internal end moments in terms of fixed end moments and near end, and far end joint rotations by slope-deflection equations.

4. Write down one equilibrium equation for each unknown joint rotation. For example, at a support in a continuous beam, the sum of all moments corresponding to an unknown joint rotation at that support must be zero. Write down as many equilibrium equations as there are unknown joint rotations.

5. Solve the above set of equilibrium equations for joint rotations.

6. Now substituting these joint rotations in the slope-deflection equations evaluate the end moments.

7. Determine all rotations.

4.2 ANALYSIS OF CONTINUOUS BEAMS

Example 4.1 Analyse the continuous beam loaded as shown below by the slope deflection method and sketch the BMD. Given $2I_{AB}=I_{BC}=2I_{CD}=2I$.

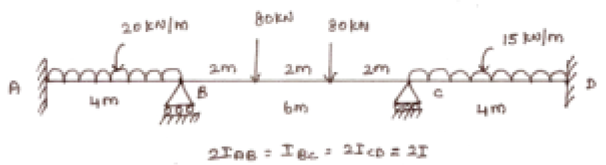


Fig. 4.2

Solution:

$I_{AB}=I_{CD}=I$, $I_{BC}=2I$, $\theta_A=\theta_D=0$(A and D are fixed)

Unknown's θ_B, θ_C

(a)Fixed end moments:

Span AB:

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \text{ kNm}$$

$$M_{FBA} = +\frac{wl^2}{12} = +\frac{20 \times 4^2}{12} = +26.67 \text{ kNm}$$

Span BC:

$$M_{FBC} = (-)\frac{Wa_1b_1^2}{l^2} + \frac{Wa_2b_2^2}{l^2} = (-)\frac{80 \times 2 \times 4^2}{6^2} + \frac{80 \times 4 \times 2^2}{6^2} = -106.67 \text{ kNm}$$

$$M_{FCB} = \frac{Wa_1b_1^2}{l^2} + \frac{Wa_2b_2^2}{l^2} = \frac{80 \times 2^2 \times 4}{6^2} + \frac{80 \times 4^2 \times 2}{6^2} = 106.67 \text{ kNm}$$

Span CD:

$$M_{FCD} = -\frac{wl^2}{12} = -\frac{15 \times 4^2}{12} = -20 \text{ kNm}$$

$$M_{FDC} = +\frac{wl^2}{12} = +\frac{15 \times 4^2}{12} = +20 \text{ kNm}$$

(b)Slope deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -26.67 + \frac{2EI}{4}(0 + \theta_B + 0) \quad (\Delta=0,\text{since no settlement of supports})$$

$$= -26.67 + 0.5\,EI\,\theta_B \quad \text{-------------------------- (1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\Delta}{l}\right]$$

$$= 26.67 + \frac{2EI}{4}(0 + 2\theta_B + 0)$$

$$= 26.67 + 0.5\,EI\,\theta_B \quad \text{--------------------------- (2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[\theta_C + 2\theta_B + \frac{3\Delta}{l}\right]$$

$$= -106.67 + \frac{2EI \times 2I}{6}(2\theta_B + \theta_C + 0)$$

$$= -106.67 + EI\,(1.333\,\theta_B + 0.666\,\theta_C) \quad \text{-------------- (3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B + \frac{3\Delta}{l}\right]$$

$$= 106.67 + \frac{2EI \times 2I}{6}(\theta_B + 2\,\theta_C + 0)$$

$$= 106.67 + EI\,(1.333\,\theta_C + 0.666\,\theta_B) \quad \text{----------------- (4)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D + \frac{3\Delta}{l}\right]$$

$$= -20 + \frac{2EI}{4}(2\theta_C + 0 + 0)$$

$$M_{CD} = -20 + EI\theta_C \qquad \text{-------------------- (5)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C + \frac{3\Delta}{l}\right]$$

$$= 20 + \frac{2EI}{4}(0 + \theta_C + 0)$$

$$M_{DC} = 20 + EI\,(0.5\theta_C) \qquad \text{-------------------- (6)}$$

(c)Equilibrium equations:

$$M_{BA} + M_{BC} = 0 \qquad \text{--------------------- (7)}$$

$$M_{CB} + M_{CD} = 0 \qquad \text{--------------------- (8)}$$

Adding (2) and (3),(7) becomes

$$26.67 + EI\,\theta_B - 106.67 + EI\,(1.333\,\theta_B + 0.666\,\theta_C) = 0$$

$$2.333\,\theta_B + 0.666\,\theta_C = 80/EI \qquad \text{--------------------- (9)}$$

Adding (4) and (5), (8) becomes

$$106.67 + EI\,(1.333\,\theta_C + 0.666\,\theta_B) - 20 + EI\,\theta_C = 0$$

$$0.666\,\theta_B + 2.333\,\theta_C = -86.67/EI \qquad \text{-------------------- (10)}$$

$$\begin{pmatrix} 2.333 & 0.666 \\ 0.666 & 2.33 \end{pmatrix} \begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = 1/EI \begin{pmatrix} 80 \\ -86.67 \end{pmatrix}$$

$$\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = 1/EI \begin{pmatrix} 48.88 \\ -51.11 \end{pmatrix}$$

(d)Final moments:

Substituting the values of θ_B and θ_C in equations (1) to (6).

$M_{AB} = -26.67 + 0.5EI\,(48.88/EI) = -2.23$ kNm

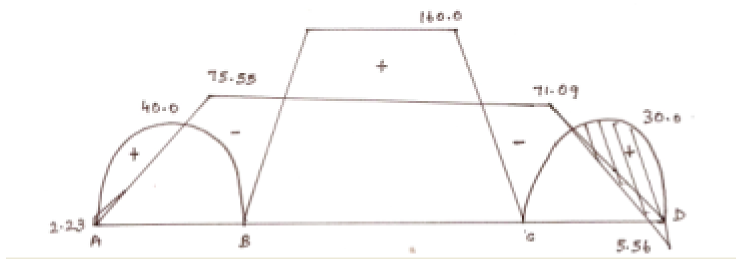
$M_{BA} = 26.67 + 0.5EI\,(48.88/EI) = 75.5$ kNm

$M_{BC} = -106.67 + EI\,(1.333\,\theta_B + 0.666\,\theta_C) = -75.55$ kNm

$M_{CB} = 106.67 + EI\,(1.333\,\theta_C + 0.666\,\theta_B) = +71.09$ kNm

$M_{CD} = -20 + EI\theta_C = -71.1$ kNm

$M_{DC} = 20 + EI\,(0.5\theta_C) = -5.56$ kNm



Bending moment diagram (BMD)

Example 4.2: Three span continuous beams ABCD is fixed at A and continuous over B, C and D. The beam subjected to loads as shown. Analyse the beam by slope deflection method and draw bending moment and shear force diagram.

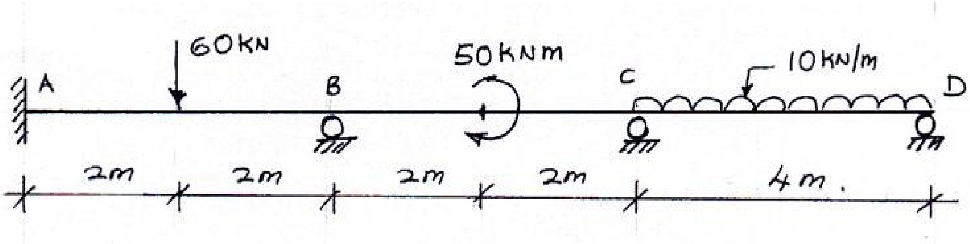


Fig. 4.3

Solution:

Since end A is fixed $\theta_A = 0, \theta_B \neq 0, \theta_c \neq 0, \theta_D = 0$

Fixed end moments:

$$F_{AB} = -\frac{Wl}{8} = -\frac{60\times4}{8} = -30\ KNM$$

$$F_{BA} = +\frac{Wl}{8} = +\frac{60\times4}{8} = +30\ KNM$$

$$F_{BC} = +\frac{M}{4} = +12.5\ KNM$$

$$F_{CB} = +\frac{M}{4} = +12.5\ KNM$$

$$F_{CD} = -\frac{wl^2}{12} = -\frac{10\times4^2}{12} = -13.33\ KNM$$

$$F_{DC} = +\frac{wl^2}{12} = +\frac{10\times4^2}{12} = +13.33\ KNM$$

Slope deflection equations:

$$M_{AB} = F_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= -30 + \frac{2EI}{4}(0 + \theta_B)$$

$$= -30 + 0.5EI\theta_B \qquad \text{-------->}(1)$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$= 30 + \frac{2EI}{4}(2\theta_B + 0)$$

$$= +30 + EI\theta_B \qquad \text{--------->}(2)$$

$$M_{BC} = F_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= 12.5 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$= 12.5 + EI\theta_B + 0.5EI\theta_C \qquad \text{--------->}(3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= 12.5 + \frac{2EI}{4}(2\theta_C + \theta_B)$$

$$= 12.5 + EI\theta_C + 0.5\,EI\theta_B \qquad \text{--------->}(4)$$

$$M_{CD} = F_{CD} + \frac{2EI}{L}(2\theta_C + \theta_D)$$

$$= -13.33 + \frac{2EI}{4}(2\theta_C + \theta_D)$$

$$= -13.33 + EI\,\theta_C + 0.5EI\,\theta_D \qquad \text{---------->}(5)$$

$$M_{DC} = F_{DC} + \frac{2EI}{L}(2\theta_D + \theta_C)$$

$$= 13.33 + \frac{2EI}{4}(2\theta_D + \theta_C)$$

$$= 13.33 + 0.5EI\,\theta_C + EI\,\theta_D \qquad \text{---------->}(6)$$

In the above Equations there are three unknowns, $EI\,\theta_B, EI\theta_C$ & $EI\theta_D$, accordingly the boundary conditions are:

i $M_{BA} + M_{BC} = 0$

ii $M_{CB} + M_{CD} = 0$

iii $M_{DC} = 0$ $(\because \text{hinged})$

Now

$$M_{BA} + M_{BC} = 0$$
$$30 + EI\theta_B + 12.5 + EI\theta_B + 0.5EI\theta_C = 0$$
$$2EI\theta_B + 0.5EI\theta_C + 42.5 = 0 \quad ------> (7)$$
$$M_{CB} + M_{BC} = 0$$
$$+12.5 + EI\theta_C + 0.5EI\theta_B - 13.33 + EI\theta_C + 0.5EI\theta_D = 0$$
$$0.5EI\theta_B + 2EI\theta_C + 0.5EI\theta_D - 0.83 = 0 \quad ------> (8)$$

$$M_{DC} = 0$$

$$13.33 + 0.5EI\theta_C + EI\theta_D = 0 \quad ------> (9)$$

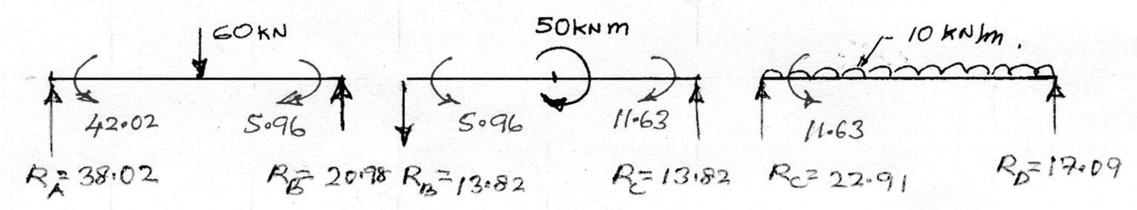
By solving (7), (8) & (9), we get

$$EI\theta_B = -24.04$$
$$EI\theta_C = +11.15$$
$$EI\theta_D = -18.90$$

By substituting the values of θ_B, θ_c and θ_D in respective equations we get

$$M_{AB} = -30 + 0.5\,(-24.04) = -42.02 \text{ KNM}$$
$$M_{BA} = +30 + (-24.04) = +5.96 \text{ KNM}$$
$$M_{BC} = +12.5 + (-24.04) + 0.5(+11.15) = -5.96 \text{ KNM}$$
$$M_{CB} = +12.5 + 11.15 + 0.5(-24.04) = +11.63 \text{ KNM}$$
$$M_{CD} = -13.33 + 11.15 + 0.5(-18.90) = -11.63 \text{ KNM}$$
$$M_{DC} = +13.33 + 0.5(11.15) + (-18.90) = 0 \text{ KNM}$$

Reactions: Consider the free body diagram of beam.



Beam AB:

$$R_B = \frac{60 \times 2 + 5.96 - 42.02}{4} = 20.985 \text{ KN}$$

$$\therefore R_A = 60 - R_B = 30.015 \text{ KN}$$

Beam BC:

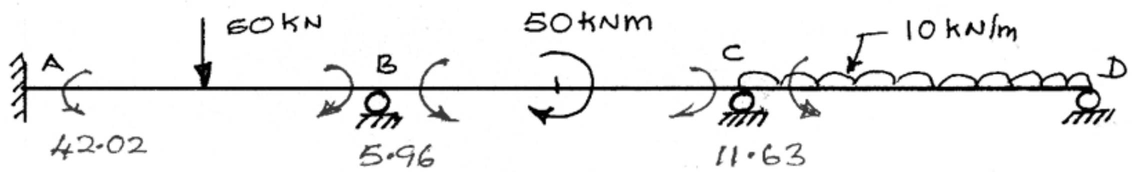
$$R_C = \frac{11.63 + 50 - 5.96}{4} = 13.92 \text{ KN}$$

$$\therefore R_B = -R_C = -13.92 \text{ KN} \quad \therefore R_B \text{ is downward}$$

Beam CD:

$$R_D = \frac{10 \times 4 \times 2 - 11.63}{4} = 17.09 \text{ KN}$$

$$\therefore R_C = 10 \times 4 - R_D = 22.91 \text{KN}$$



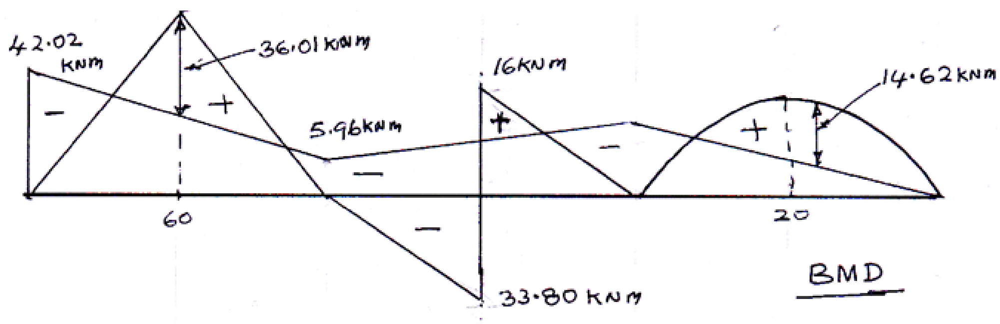
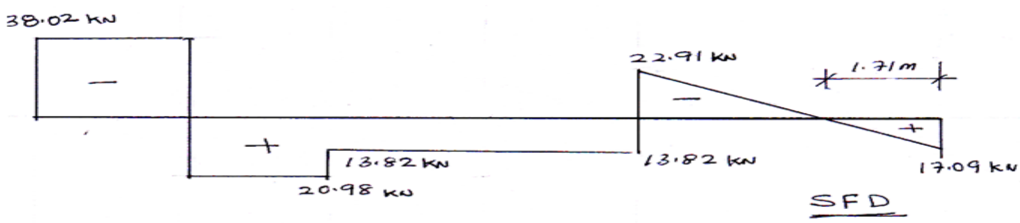


Fig. 4.6 bending moment diagram



Example: 4.3 Analyse the continuous beam shown using slope deflection method. Then draw bending moment and shear force diagram.

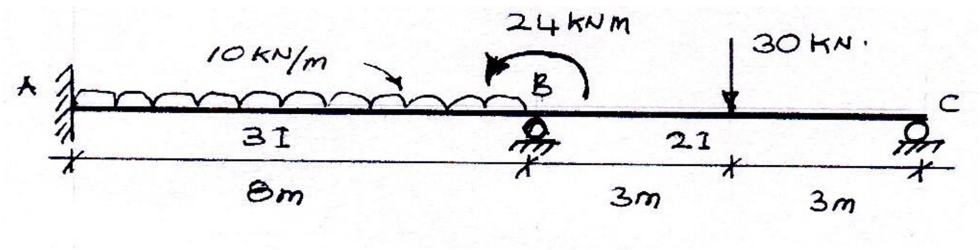


Fig. 4.4

Solution: In this problem $\theta_A = 0$, $\because$ end A is fixed

FEMs:

$$F_{AB} = -\frac{wl^2}{12} = -\frac{10 \times 8^2}{12} = -53.33 \text{ KNM}$$

$$F_{BA} = +\frac{wl^2}{12} = +53.33 \text{ KNM}$$

$$F_{BC} = -\frac{Wl}{8} = -\frac{30 \times 6}{8} = -22.50 \text{ KNM}$$

$$F_{CD} = +\frac{WL}{8} = +22.50 \text{ KNM}$$

Slope deflection equations:

$$M_{AB} = F_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= -53.33 + \frac{2E \times 3I}{8}(0 + \theta_B)$$

$$= -53.33 + \frac{3}{4}EI\theta_B \qquad \text{--------> (1)}$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$= +53.33 + \frac{2E \times 3I}{8}(2\theta_B + 0)$$

$$= 53.33 + \frac{3}{2}EI\theta_B \qquad \text{--------> (2)}$$

$$M_{BC} = F_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -22.5 + \frac{2E2I}{6}(2\theta_B + \theta_C)$$

$$= -22.5 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C \quad \text{--------} > (3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= +22.5 + \frac{2E2I}{6}(2\theta_C + \theta_B)$$

$$= +22.5 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B \quad \text{--------} > (4)$$

In the above equation there are two unknown θ_B and θ_C, accordingly the boundary conditions are:

$$\text{i } -M_{BA} - M_{BC} - 24 = 0$$
$$\text{ii } M_{CB} = 0$$

$$\text{Now, } M_{BA} + M_{BC} - 24 = 53.33 + \frac{3}{2}EI\theta_B - 22.5 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C + 24$$

$$= +54.83 + \frac{17}{6}EI\theta_B + \frac{2}{3}EI\theta_C = 0 \quad \text{------} > (5)$$

$$\text{and } M_{CB} = 22.5 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B = 0$$

$$\therefore \frac{2}{3}EI\theta_C = -11.25 - \frac{1}{3}EI\theta_B \quad \text{-----------} > (6)$$

Substituting in eqn. (5)

$$54.83 + \frac{17}{6}EI\theta_B - 11.25 - \frac{1}{3}EI\theta_B = 0$$

$$+44.58 + \frac{15}{6}EI\theta_B = 0$$

$$\therefore EI\theta_B = -\frac{44.58 \times 6}{15} = -17.432 \quad \text{rotation anticlockwise}$$

$\therefore$ from equation (6)

$$EI\theta_C = \frac{3}{2}\left[-11.25 - \frac{1}{3}(-17.432)\right]$$

$$= -8.159 \quad \text{rotation anticlockwise}$$

Substituting $EI\theta_B = -17.432$ and $EI\theta_C = -8.159$ in the slope deflection equation we get

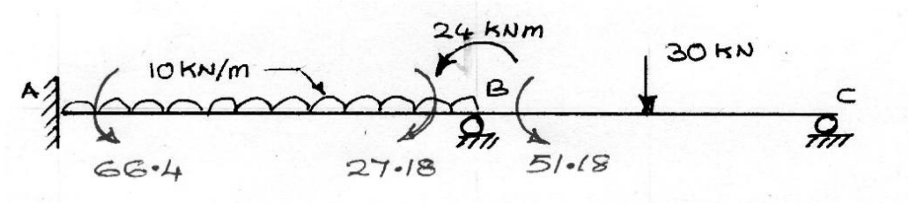
Final Moments:

$$M_{AB} = -53.33 + \frac{3}{4}(-17.432) = -66.40 \quad \text{KNM}$$

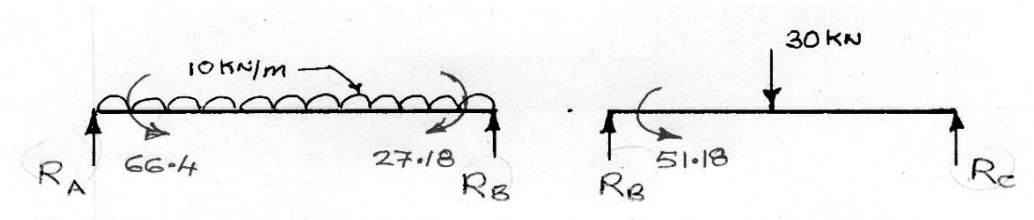
$$M_{BA} = +53.33 + \frac{3}{2}(-17.432) = +27.18 \quad \text{KNM}$$

$$M_{BC} = -22.5 + \frac{4}{3}(-17.432) + \frac{2}{3}(-8.159) = -51.18 \quad KNM$$

$$M_{CB} = +22.5 + \frac{4}{3}(-8.159) + \frac{2}{3}(-17.432) = 0.00$$



<u>Reactions:</u> Consider free body diagram of beams as shown



Span AB:

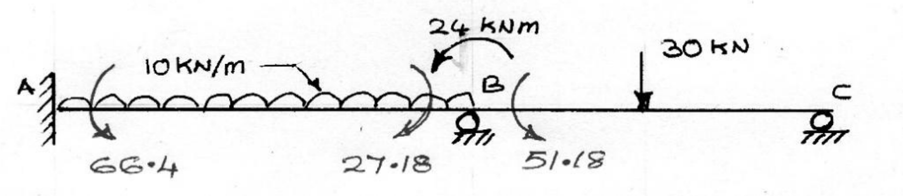
$$R_B = \frac{27.18 - 66.40 + 10 \times 8 \times 4}{8} = 35.13 \ KN$$

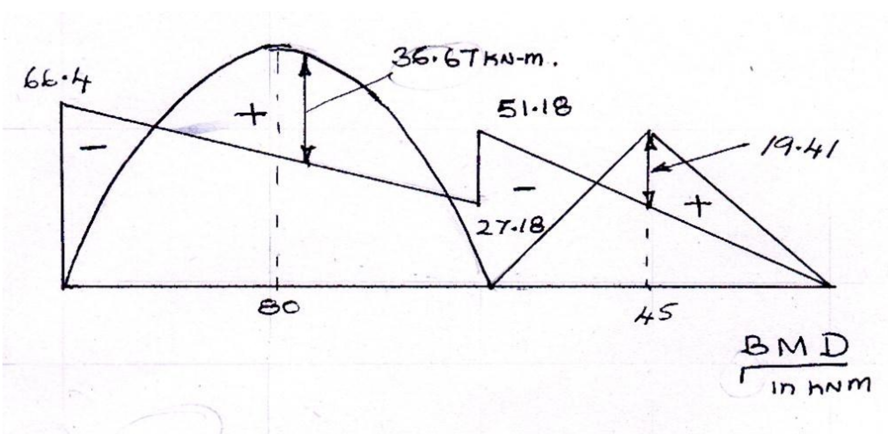
$$\therefore R_A = 10 \times 8 - R_B = 44.87 \ KN$$

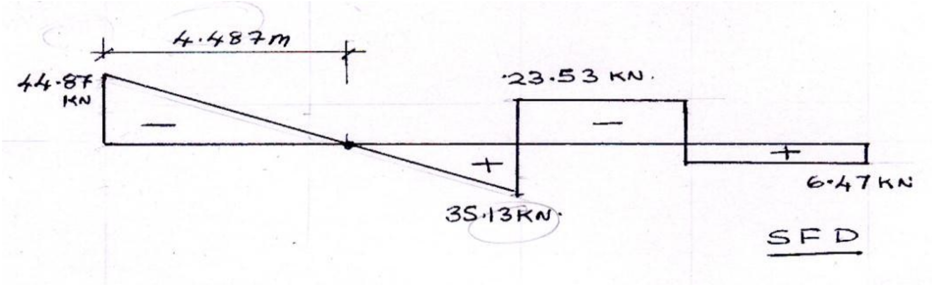
Span BC:

$$R_B = \frac{51.18 + 30 \times 3}{6} = 23.53 \ KN$$

$$R_C = 30 - R_B = 6.47 \quad KN$$







Max BM

Span AB: Max BM occurs where SF=0, consider SF equation with A as origin

$$S_x = 44.87 - 10x = 0$$
$$x = 4.487\,m$$
$$\therefore M_{max} = 44.87 \times 4.487 - 10 \times \frac{4.487^2}{2} - 64 = 36.67\,KNM$$

Span BC: Max BM occurs under point load

$$BC\ M_{max} = 45 - \frac{51.18}{2} = 19.41\,KN\,M$$

Example:4.4 Analyse the beam shown in figure. End support C is subjected to an anticlockwise moment of 12 kNm.

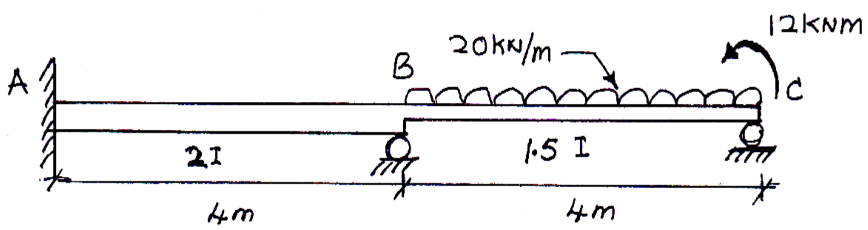


Fig. 4.5

Solution:

In this problem $\theta_A = 0,\ \therefore$ end is fixed

FEMs:

$$F_{BC} = -\frac{wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \quad KNM$$

$$F_{CB} = +\frac{wl^2}{12} = +\ 26.67 \quad KNM$$

Slope deflection equations:

$$M_{AB} = F_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= 0 + \frac{2E2I}{4}(0 + \theta_B)$$

$$= EI\theta_B \qquad \text{---------} > (1)$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$= 0 + \frac{2E2I}{4}(2\theta_B + 0)$$

$$= 2EI\theta_B \qquad \text{---------} > (2)$$

$$M_{BC} = F_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -26.67 + \frac{2E \times 1.5I}{4}(2\theta_B + \theta_C)$$

$$= -26.67 + \frac{3}{2}EI\theta_B + \frac{3}{4}EI\theta_C \qquad \text{---------} > (3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= +26.67 + \frac{2E \times 1.5I}{4}(2\theta_C + \theta_B)$$

$$= +26.67 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_B \qquad \text{---------} > (4)$$

In the above equation there are two unknowns θ_B and θ_C , accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + 12 = 0$$

$$\text{Now, } M_{BA} + M_{BC} = 2EI\theta_B - 26.67 + \frac{3}{2}EI\theta_B + \frac{3}{4}EI\theta_C$$

$$= \frac{7}{2}EI\theta_B + \frac{3}{4}EI\theta_C - 26.67 = 0 \qquad \text{---------} > (5)$$

$$\text{and, } M_{CB} + 12 = 26.67 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_B + 12$$

$$= 38.67 + \frac{3}{4}EI\theta_B + \frac{3}{2}EI\theta_C = 0 \qquad \text{---------} > (6)$$

$$\frac{7}{2}EI\theta_B + \frac{3}{4}EI\theta_C - 26.67 = 0$$

From (5) and (6)

$$\frac{3}{8}EI\theta_B + \frac{3}{4}EI\theta_C + 19.33 = 0$$

$$\frac{25}{8}EI\theta_B - 46 = 0$$

$$EI\theta_B = +46 \times \frac{8}{25} = +14.72$$

From (6)

$$EI\theta_C = -\frac{2}{3}\left(38.67 + \frac{3}{4}(14.72)\right)$$

$$= -33.14 \qquad \text{-ve signindicates rotation anticlockwis e}$$

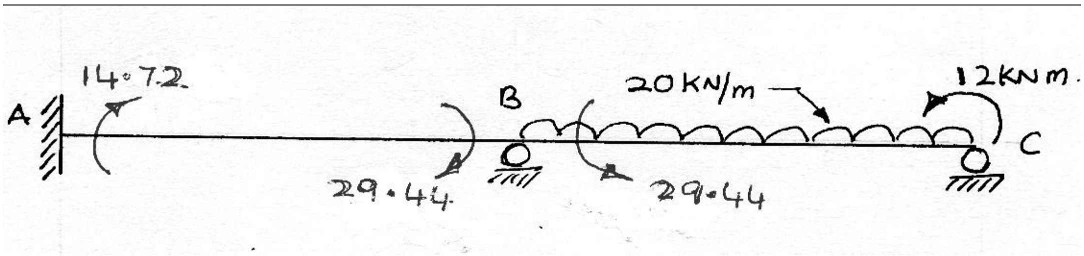
Substituting $EI\theta_B$ and $EI\theta_C$ is slope deflection equations

$$M_{AB} = EI\theta_B = +14.72 \text{ KNM}$$

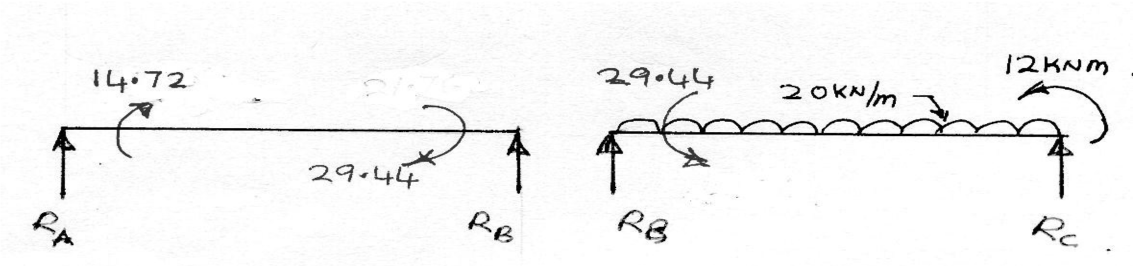
$$M_{BA} = 2EI\theta_B = 2(14.72) = 29.42 \text{ KNM}$$

$$M_{BC} = -26.67 + \frac{3}{2}(14.72) + \frac{3}{4}(-33.14) = 29.44 \text{ KNM}$$

$$M_{CB} = +26.67 + \frac{3}{2}(-33.14) + \frac{3}{4}(14.72) = -12 \text{ KNM}$$



Reaction: Consider free body diagrams of beam



Span AB:

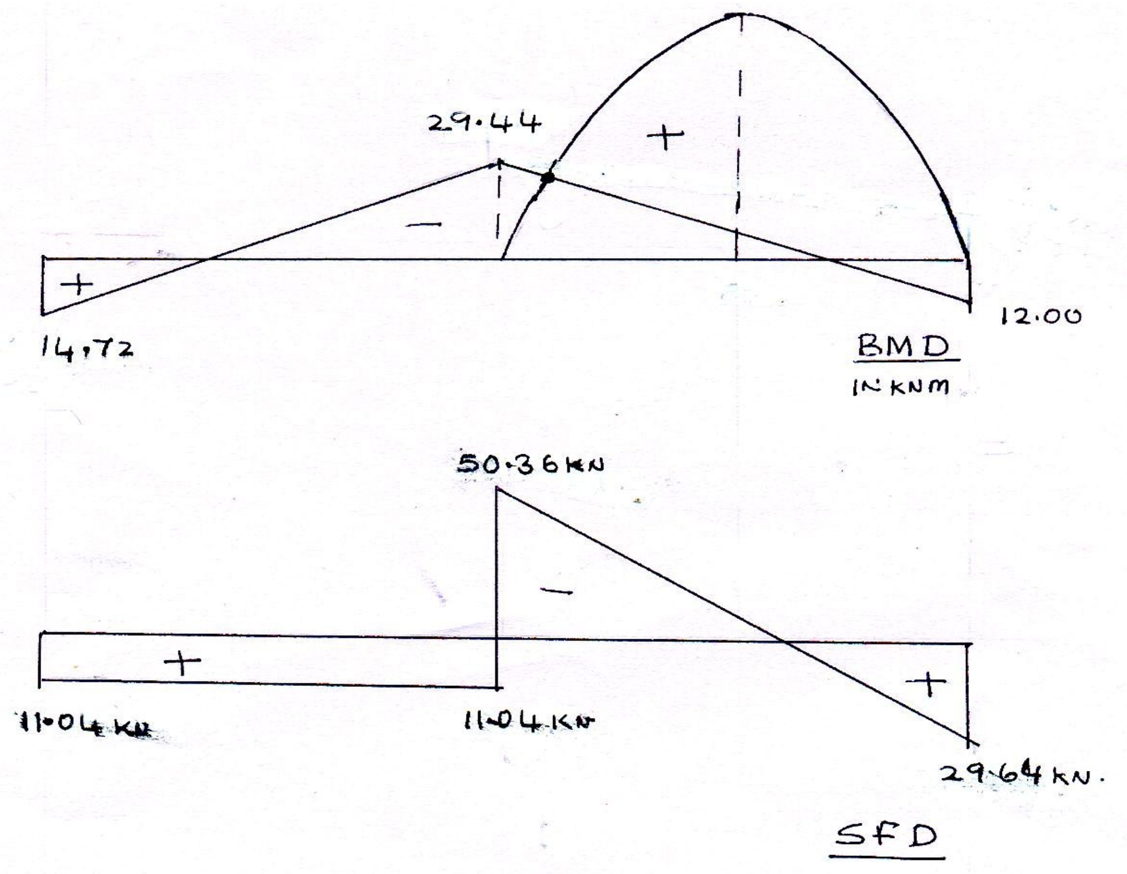
$$R_B = \frac{14.72 + 29.44}{4} = 11.04 \text{ KN}$$

$$R_A = -R_B = -11.04 \text{ KN}$$

Span BC:

$$R_B = \frac{29.44 + 12 + 20 \times 4 \times 2}{4} = 50.36 \text{ KN}$$

$$R_C = 20 \times 4 - R_B = 29.64 \text{ KN}$$



4.3 RIGID FRAMES WITHOUT SWAY (ANTISYMMETRY)

Example: 4.5 Analyse the simple frame shown in figure. End A is fixed and ends B & C are hinged. Draw the bending moment diagram.

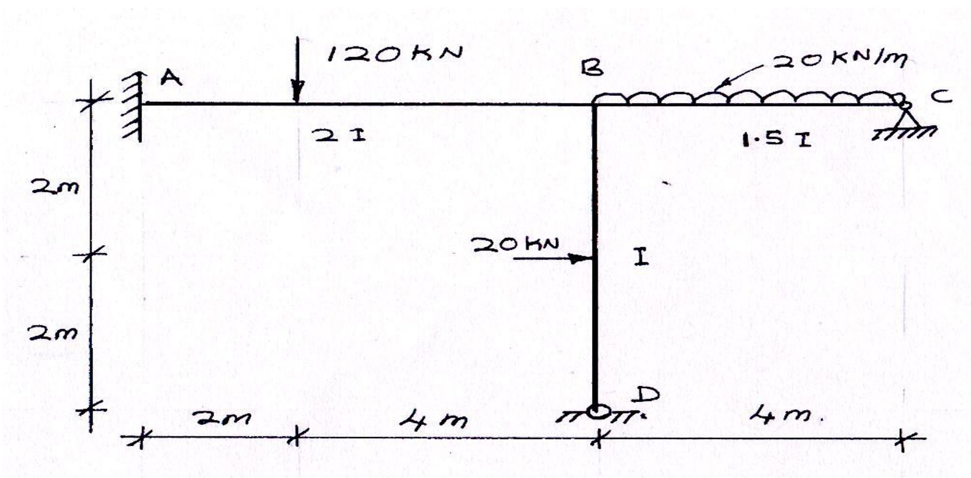


Fig. 4.6

Solution:

In this problem $\theta_A = 0, \theta_B \neq 0, \theta_C \neq 0, \theta_D \neq 0,$

Fixed end moments:

$$F_{AB} = -\frac{Wab^2}{L^2} = -\frac{120 \times 2 \times 4^{\,2}}{6^2} = -106.67\ \text{KNM}$$

$$F_{BA} = +\frac{Wa^2b}{L^2} = +\frac{120 \times 2^2 \times 4}{6^2} = +53.33\ \text{KNM}$$

$$F_{BC} = -\frac{wl^2}{12} = -\frac{20 \times 4^{\,2}}{12} = -26.67\ \text{KNM}$$

$$F_{CB} = +\frac{wl^2}{12} = +\frac{20 \times 4^{\,2}}{12} = +26.67\ \text{KNM}$$

$$F_{CD} = +\frac{WL}{8} = +\frac{20 \times 4}{8} = +10\ \text{KNM}$$

$$F_{DB} = -\frac{WL}{8} = -10\ \text{KNM}$$

Slope deflections are

$$M_{AB} = F_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= -106.67 + \frac{2E2I}{6}(\theta_B) = -106.67 + \frac{2}{3}EI\theta_B \qquad ----> (1)$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_B)$$

$$= +53.33 + \frac{2E2I}{6}(2\theta_B) = +53.33 + \frac{4}{3}EI\theta_B \qquad ----> (2)$$

$$M_{BC} = F_{CB} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -26.67 + \frac{2E}{4} \times \frac{3I}{2}(2\theta_B + \theta_C) = -26.67 + \frac{3}{2}EI\theta_B + \frac{3}{4}EI\theta_C \qquad ----> (3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= +26.67 + \frac{2E}{4} \times \frac{3I}{2}(2\theta_C + \theta_B) = +26.67 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_B \qquad ----> (4)$$

$$M_{BD} = F_{BD} + \frac{2EI}{L}(2\theta_B + \theta_D)$$

$$= +10 + \frac{2EI}{4}(2\theta_B + \theta_D) = +10 + EI\theta_B + \frac{1}{2}EI\theta_D \qquad ----> (5)$$

$$M_{DB} = F_{DB} + \frac{2EI}{L}(2\theta_D + \theta_B)$$

$$= -10 + \frac{2EI}{4}(2\theta_D + \theta_B) = -10 + EI\theta_D + \frac{1}{2}EI\theta_B \qquad ----> (6)$$

In the above equations we have three unknown rotations θ_B , θ_C , θ_D accordingly we have three boundary conditions.

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$M_{CB} = 0 \qquad \text{Since C and D are hinged}$$

$$M_{DB} = 0$$

Now

$$M_{BA} + M_{BC} + M_{BD} = 53.33 + \frac{4}{3}EI\theta_B - 26.67 + \frac{3}{2}EI\theta_B + \frac{3}{4}EI\theta_C + 10 + EI\theta_B + \frac{1}{2}EI\theta_D$$

$$= 36.66 + \frac{23}{6}EI\theta_B + \frac{3}{4}EI\theta_C + \frac{1}{2}EI\theta_D = 0 \qquad ----> (7)$$

$$M_{CB} = 26.67 + \frac{3}{4}EI\theta_B + \frac{3}{2}EI\theta_C = 0 \qquad ----> (8)$$

$$M_{DB} = -10 + \frac{1}{2}EI\theta_B + EI\theta_D = 0 \qquad ----> (9)$$

Solving equations 7, 8, & 9 we get

$$EI\theta_B = -8.83$$
$$EI\theta_C = -13.36$$
$$EI\theta_D = +14.414$$

Substituting these values in slope equations

$$M_{AB} = -106.67 + \frac{2}{3}(-8.83) = -112.56\ KNM$$

$$M_{BA} = 53.33 + \frac{4}{3}(-8.83) = 41.56\ KNM$$

$$M_{BC} = -26.67 + \frac{3}{2}(-8.3) + \frac{3}{4}(-13.36) = -49.94\ KNM$$

$$M_{CB} = +26.67 + \frac{3}{2}(-13.36) + \frac{3}{4}(-8.83) = 0$$

$$M_{BD} = 10 + (-8.83) + \frac{1}{2}(+14.414) = 8.38\ kNm$$

$$M_{DB} = -10 + (14.414) + \frac{1}{2}(-8.83) = 0$$

Reactions: Consider free body diagram of each members

Span AB:

$$R_B = \frac{41.56 - 112.56 + 120 \times 2}{6} = 28.17 \text{ KN}$$

$$\therefore R_A = 120 - R_B = 91.83 \text{ KN}$$

Span BC:

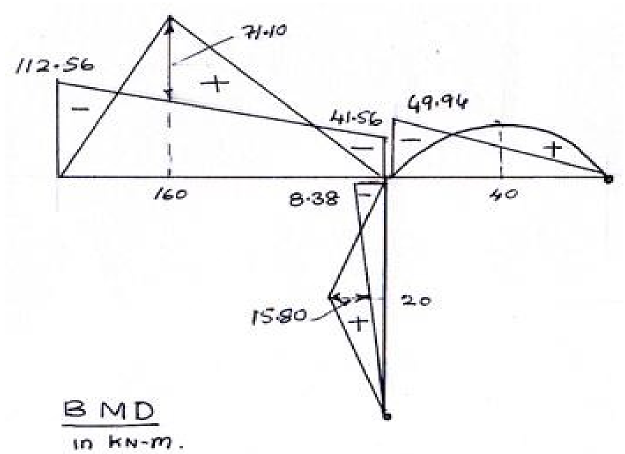
$$R_B = \frac{49.94 + 20 \times 4 \times 2}{4} = 52.485 \text{ KN}$$

$$\therefore R_C = 20 \times 4 - R_B = 27.515 \text{ KN}$$

Column BD:

$$H_D = \frac{20 \times 2 - 8.33}{4} = 7.92 \text{ KN}$$

$$\therefore H_B = 12.78 \text{ KN} \qquad [\because H_A + H_D = 20]$$



4.4 RIGID FRAMES WITHOUT SWAY (Symmetry)

Example: 4.6 Analyse the portal frame shown in figure and also drawn bending moment and shear force diagram.

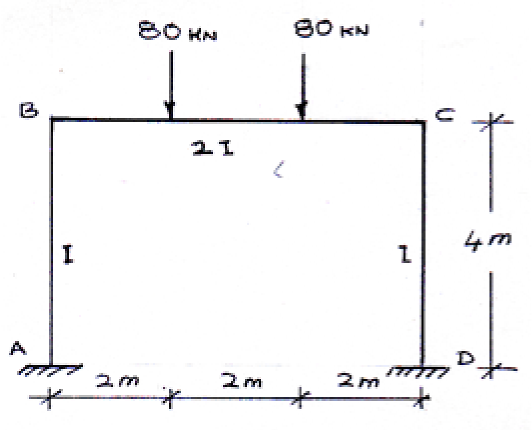


Fig. 4.7

Solution:

Symmetrical problem

- Sym frame + Sym loading

$\theta_A = 0, \quad \theta_B \neq 0, \quad \theta_C \neq 0, \quad \theta_D = 0$

Fixed end moments:

$$F_{BC} = -\frac{W_1 ab^2}{L^2} - \frac{W_2 cd^2}{L^2}$$

$$= \frac{80 \times 2 \times 4^2}{6^2} - \frac{80 \times 4 \times 2^2}{6^2} = \quad -106.67 \ \text{KNM}$$

$$F_{CB} = +\frac{Wa^2b}{L^2} + \frac{W_2c^2d}{L^2} = +106.67 \ \text{KNM}$$

Slope deflection equations:

$$M_{AB} = F_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B) = 0 + \frac{2EI}{4}(0 + \theta_B) = \frac{1}{2}EI\theta_B \qquad \text{-------->}(1)$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A) = 0 + \frac{2EI}{4}(2\theta_B + 0) = EI\theta_B \qquad \text{------->}(2)$$

$$M_{BC} = F_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -106.67 + \frac{2E2I}{6}(2\theta_B + \theta_C) = -106.67 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C \quad \text{------>}(3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= +106.67 + \frac{2E2I}{6}(2\theta_C + \theta_B) = +106.67 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B \quad \text{------>}(4)$$

$$M_{CD} = F_{CD} + \frac{2EI}{L}\left(2\theta_C + \theta_D\right)$$

$$= 0 + \frac{2EI}{4}(2\theta_C + 0) = EI\theta_C \qquad \text{------->}(5)$$

$$M_{DC} = F_{DC} + \frac{2EI}{L}\left(2\theta_D + \theta_C\right)$$

$$= 0 + \frac{2EI}{4}(0 + \theta_C) = \frac{1}{2}EI\theta_C \qquad \text{------->}(6)$$

In the above equation there are two unknown rotations. Accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

Now $M_{BA} + M_{BC} = -106.67 + \frac{7}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0 \qquad \text{------->}(7)$

$$M_{CB} + M_{CD} = +106.67 + \frac{2}{3}EI\theta_B + \frac{7}{3}EI\theta_C = 0 \qquad \text{------->}(8)$$

Multiply by (7) and (8) by 2

$$\left.\begin{array}{l} -746.69 + \frac{49}{3}EI\theta_B + \frac{14}{3}EI\theta_C = 0 \\ +213.34 + \frac{4}{3}EI\theta_B + \frac{14}{3}EI\theta_C = 0 \end{array}\right\} \text{subtracts}$$

$$-960.03 + \frac{45}{3}EI\theta_B = 0$$

$$EI\theta_B = +960.03 \times \frac{3}{45} = +\ 64 \quad \text{Clockwise}$$

Using equation (7)

$$EI\theta_C = -\frac{3}{2}\left[-106.67 + \frac{7}{3}EI\theta_B\right]$$

$$= -\frac{3}{2}\left[-106.67 + \frac{7}{3}\times 64\right] = -\ 64 \quad \text{Anticlo ckwise}$$

Here we find $\theta_B = -\ \theta_C$. It is obvious because the problem is symmetrical.

$\therefore$ Final moments are

$$M_{AB} = +\frac{64}{2} = +32 \text{ KNM}$$

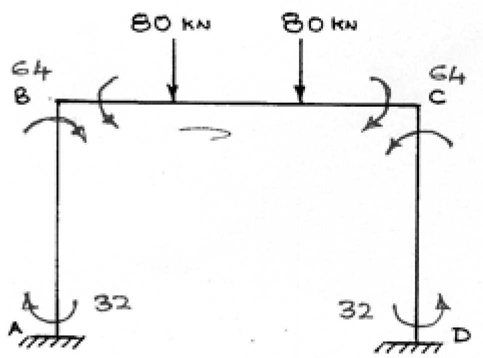
$$M_{BA} = 64 \text{ KNM}$$

$$M_{BC} = -106.67 + \frac{4}{3}64 + \frac{2}{3}(-64) = -64 \text{ KNM}$$

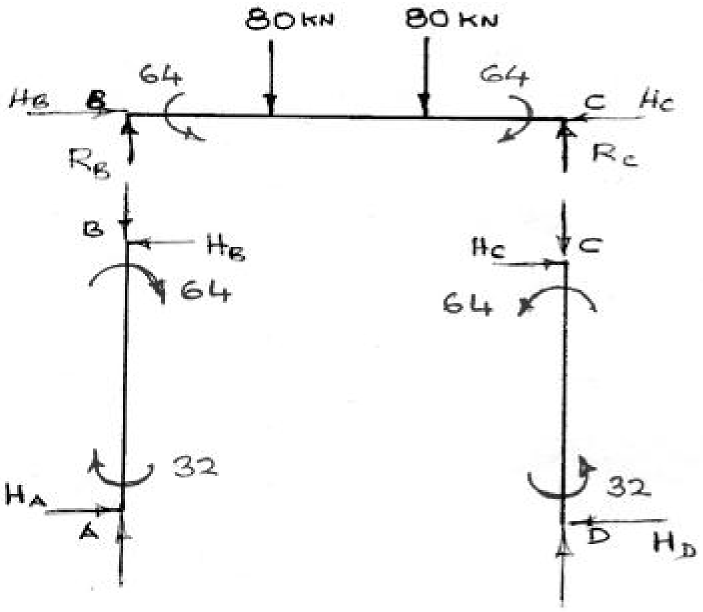
$$M_{CB} = +106.67 + \frac{4}{3}(-64) + \frac{2}{3}(64) = +64 \text{ KNM}$$

$$M_{CD} = -64 \text{ KNM}$$

$$M_{DC} = -\frac{1}{2}64 = -32 \text{ KNM}$$



Consider free body diagram's of beam and columns as shown



By symmetrical we can write

$$R_A = R_B = 60 \text{ KNM}$$

$$R_D = R_C = 80 \text{ KNM}$$

Now consider free body diagram of column AB,Apply

$$\sum M_B = 0$$

$$H_A \times 4 = 64 + 32$$

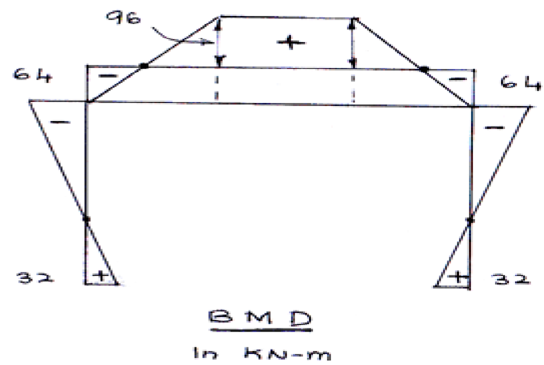
$$\therefore\ H_A = 24 \quad KN$$

Similarly from free body diagram of column CD, Apply

$$\sum M_C = 0$$

$$H_A \times 4 = 64 + 32$$

$$\therefore\ H_D = 24 \quad KN$$



Example 4.7 Analyse the portal frame loaded as shown in figure by the slope deflection method and sketch the bending moment and shear force diagrams.

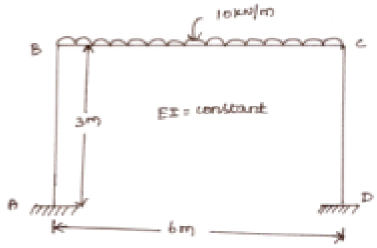


Fig. 4.8

Solution:

(a)Fixed end moments:

Span BA: $M_{FAB} = M_{FBA} = 0$

Span BC: $M_{FBC} = -\dfrac{wl^2}{12} = -\dfrac{10 \times 6^2}{12} = -30$ kNm

$M_{FCB} = +\dfrac{Wl^2}{12} = -\dfrac{10 \times 6^2}{12} = +30$ kNm

Span CD: $M_{FCD} = M_{FDC} = 0$

(b)Slope deflection equations:

The frame is unsymmetrical. Hence the frame will sway. Let B and C move horizontally by δ.

Span AB:

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\delta}{l}\right]$$

$$=2/3EI\,\theta_B \qquad \text{----------------} \; (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\delta}{l}\right]$$

$$= \frac{4EI}{3}\theta_B \qquad \text{----------------} \; (2)$$

Span BC:

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[\theta_c + 2\theta_B + \frac{3\delta}{l}\right]$$

$$M_{BC} = -30 + \frac{EI}{3}\theta_B \qquad \text{----------------} \; (3)$$

(C)Joint Equilibrium Equation:

$$M_{BA} + M_{BC} = 0$$

From (2) and (3), $4EI/3\;\theta_B\;-30+EI/3\;\theta_B$

$$\theta_B = 18/EI$$

$$\theta c = -\theta_B = -18/EI$$

(d)Final moments:

Span AB:

$$M_{AB} = \frac{2EI}{3}(18/EI)$$

$$M_{AB} = 12\text{ kNm}$$

$$M_{BA} = \frac{4EI}{3}(18/EI)$$

$$M_{BA} = 24\text{ kNm}$$

Span BC:

$$M_{BC} = -30 + \frac{EI}{3}(18/EI) = -24\text{ kNm}$$

M_{CB} = 24 kNm

Span CD:

M_{CD} = -24 kNm

M_{DC} = -12 kNm

(e)To draw S.F.D.:

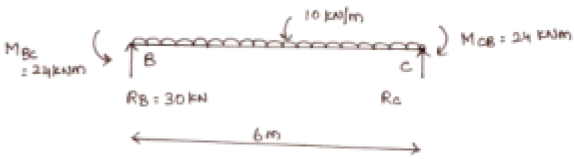
Span AB:

Taking moments about A,

$$M_{AB}+M_{BA} - F_{BA}(3) = 0$$

$$F_{BA}= 12 \text{ kN}= F_{AB}$$

Span BC:



Taking moments about B,

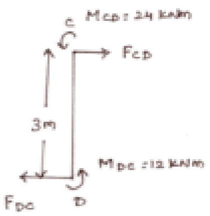
$R_B= R_C = 10X6/2=30KN$

$R_C = 25.484$ kN

R_B =Total load – Rc

= 50 – 25.484 = 24.516 kN

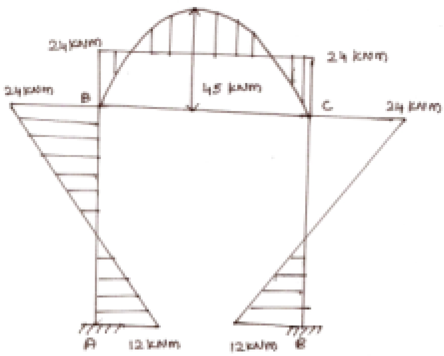
Span CD:



Taking moments about C,

$-M_{CD}+H_{DC}(4) = 0$

$H_{DC} = H_{CD} = 4.019$ kN



Bending Moment Diagram

4.5 RIGID FRAMES WITH SIDE SWAY (antisymmetry)

Portal frames may sway due to one of the following reasons.

1. Eccentric or unsymmetrical loading on the frames.
2. Unsymmetrical shape of the frames.
3. Different end conditions of the columns of the portal frames.
4. Non uniform section of the members of the frame.
5. Horizontal loading on the columns of the frame.
6. Settlement of the supports of the frame.
7. A combination of the above.

Example 4.8 Analyse the portal frame loaded as shown in figure by the slope deflection method and sketch the bending moment and shear force diagram.

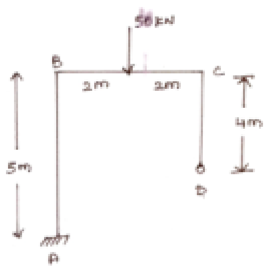


Fig. 4.9

Solution:

(a)Fixed end moments:

Span AB, CD: $M_{FAB}=M_{FBA}=M_{FDC}=M_{FCD}=0$

Span BC:

$$M_{FBC} = -\frac{wl}{8} = -\frac{50 \times 4}{8} = -25 \text{ kNm}$$

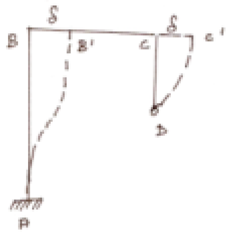
wl 50×4

$$M_{FBC} = + \frac{\;}{8} = - \frac{\;}{8} = +25 \text{ kNm}$$

(b)Slope deflection equations:

The frame is unsymmetrical. Hence the frame will sway. Let B and C move horizontally by δ.

Span AB:



$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{5}\left[2(0) + \theta_B - \frac{3\delta}{l}\right]$$

$$= \frac{2EI}{5}(\theta_B - 3\delta/5) \qquad \text{---------------- (1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\delta}{l}\right]$$

$$= \frac{2EI}{5}(2\theta_B - 3\delta/5) \qquad \text{---------------- (2)}$$

Span BC:

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[\theta_C + 2\theta_B + \frac{3\delta}{l}\right]$$

$$M_{BC} = -25 + \frac{EI}{2}(\theta_C + 2\theta_B) \qquad \text{---------------- (3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B + \frac{3\delta}{l}\right]$$

EI

$$M_{CB} = 25 + \frac{}{2} (2\theta_C + \theta_B) \qquad \text{----------------} (4)$$

Span CD:

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D + \frac{3\delta}{l}\right]$$

$$M_{CD} = \frac{EI}{2} (2\theta_C + \theta_D - 3\delta/4) \qquad \text{----------------} (5)$$

$$M_{DC} = M_{FCD} + \frac{2EI}{l}\left[\theta_C + 2\theta_D + \frac{3\delta}{l}\right]$$

$$M_{DC} = \frac{EI}{2} (\theta_C + 2\theta_D - 3\delta/4) \qquad \text{----------------} (6)$$

(c)Shear equation:

$EI (0.24\,\theta_B - 0.096\delta + 0.375\theta_C + 0.375\theta_D - 0.1875\delta) = 0$

$EI (0.24\,\theta_B + 0.375\theta_C + 0.375\theta_D - 0.248\delta) = 0$

Guass seidel iteration method:

	θ_B	θ_C	θ_D	Δ
Initial values	0	0	0	0
	13.89	-15.97	7.99	4.79
	18.96	-18.34	10.97	6.30
	19.82	-19.02	11.87	7.32
	20.14	-19.13	12.31	8.03
	20.27	-19.14	12.58	8.48
	20.33	-19.14	12.75	8.76
	20.37	-19.14	12.85	8.93
	20.39	-19.14	12.92	9.03
	20.40	-19.14	12.96	9.10
	20.41	-19.14	12.98	9.14
	20.42	-19.14	12.99	9.16
	20.42	-19.14	13.00	9.17
	20.42	-19.14	13.01	9.18
	20.42	-19.14	13.01	9.19

	20.43	-19.14	13.01	9.19
	20.43	-19.14	13.01	9.19

$\theta_B = 20.43/EI$

$\theta_C = - 19.14/EI$

$\theta_D = 13.01/EI$

$\delta = 9.19/EI$

(d)Final moments:

$$M_{AB} = \frac{2EI}{5} (\theta_B - 3\delta/5)$$

$$M_{AB} = 5.996 \text{ kNm}$$

$$M_{BA} = \frac{2EI}{5} (2\theta_B - 3\delta/5)$$

$$M_{BA} = 14.138 \text{ kNm}$$

$$M_{BC} = - 25 + \frac{EI}{2} (\theta_C + 2\theta_B) = -14.138 \text{ kNm}$$

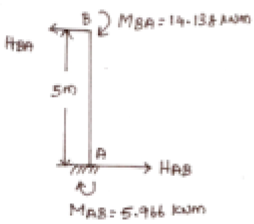
$$M_{CB} = 25 + \frac{EI}{2} (2\theta_C + \theta_B) = 16.075 \text{ kNm}$$

$$M_{CD} = \frac{EI}{2} (2\theta_C + \theta_D - 3\delta/4) = -16.075 \text{ kNm}$$

$$M_{DC} = \frac{EI}{2} (\theta_C + 2\theta_D - 3\delta/4) = 0 \text{ kNm}$$

(e)To draw S.F.D.:

Span AB:



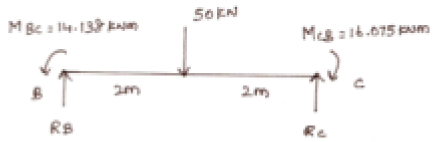
Taking moments about A,

$$M_{AB}+M_{BA} – H_{BA} (5) = 0$$

$5.966+14.138 - H_{BA}(5) = 0$

$H_{BA} = 4.021\ kN = H_{AB}$

Span BC:



Taking moments about B,

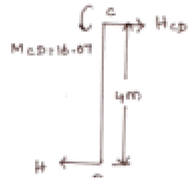
$M_{CB}+50(2)-M_{BC} - R_C(4) = 0$

$R_C = 25.484\ kN$

R_B =Total load – Rc

$= 50 - 25.484 = 24.516\ kN$

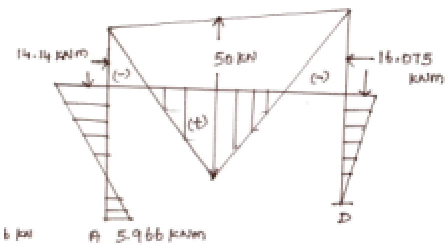
Span CD:



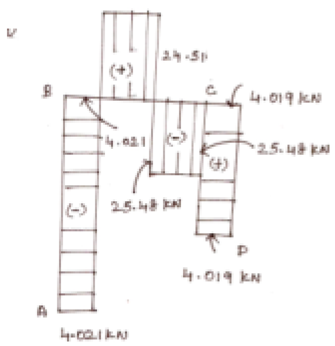
Taking moments about C,

$-M_{CD}+H_{DC}(4) = 0$

$H_{DC} = H_{CD} = 4.019\ kN$



Bending moment moment



Shear force diagram

4.6 RIGID FRAMES WITH SIDE SWAY (SYMMETRY)

Example 4.9 Analyse the portal frame subjected to loads as shown. Also draw bending moment diagram.

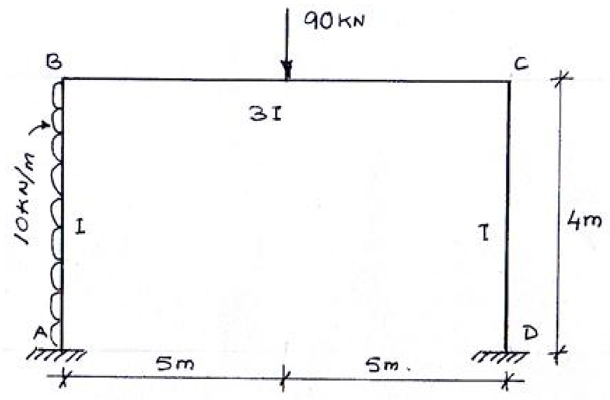


Fig. 4.10

The frame is symmetrical but loading is unsymmetrical. Hence there is a sway.

Assume sway towards right. In this problem $\theta_A = 0, \theta_B \neq 0, \theta_C \neq 0, \theta_D = 0$

FEMs:

$$F_{AB} = -\frac{wl^2}{12} = -\frac{10\times 4^2}{12} = -13.33 \text{ KNM}$$

$$F_{BA} = +\frac{wl^2}{12} = +\frac{10\times 4^2}{12} = +13.33 \text{ KNM}$$

$$F_{BC} = -\frac{wl}{8} = -\frac{90\times 10}{8} = -112.5 \text{ KNM}$$

$$F_{CB} = \frac{wl}{8} = \frac{90\times 10}{8} = +112.5 \text{ KNM}$$

Slope deflection equations:

$$M_{AB} = F_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$= -13.33 + \frac{2EI}{4}\left(0 + \theta_B - \frac{3\delta}{4}\right)$$

$$= -13.33 + 0.5\,EI\theta_B - 0.375\,EI\delta \qquad \text{--------} > (1)$$

$$M_{BA} = F_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A + \frac{3\delta}{L}\right)$$

$$= 13.33 + \frac{2EI}{4}\left(2\theta_B + 0 - \frac{3\delta}{4}\right)$$

$$= 13.33 + EI\theta_B - 0.375\,EI\delta \qquad \text{---------} > (2)$$

$$M_{BC} = F_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -112.5 + \frac{2E3I}{10}(2\theta_B + \theta_C)$$

$$= -112.5 + 1.2EI\theta_B + 0.6EI\theta_C \qquad \text{---------} > (3)$$

$$M_{CB} = F_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= +112.5 + \frac{2E3I}{10}(2\theta_C + \theta_B)$$

$$= 112.5 + 1.2EI\theta_C + 0.6\ EI\theta_B \qquad \text{---------} > (4)$$

$$M_{CD} = F_{CD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_C + 0 - \frac{3\delta}{4}\right)$$

$$= EI\,\theta_C - 0.375EI\delta \qquad \text{---------} > (5)$$

$$M_{DC} = F_{DC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{4}\left(0 + 2\theta_C - \frac{3\delta}{4}\right)$$

$$= 0.5EI\,\theta_C - 0.375EI\delta \qquad \text{---------} > (6)$$

There are 3 unknowns $EI\theta_{B,}$ $EI\theta_C$ and $EI\delta$, accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$
$$H_A + H_D + 40 = 0$$

$$\text{Here } H_A \times 4 = M_{AB} + M_{BA} - 10 \times 4 \times \frac{4}{2}$$

$$H_A = \frac{M_{AB} + M_{BA} - 80}{4}$$

$$\text{and } H_D \times 4 = M_{CD} + M_{BC}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\therefore \frac{M_{AB} + M_{BA} - 80}{4} + \frac{M_{CD} + M_{DC}}{4} + 40 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 80 = 0$$

Now $M_{BA} + M_{BC} = 0$

$$13.33EI\theta_B - 0.375EI\delta - 112.5 + 1.2EI\theta_B + 0.6EI\theta_C = 0$$

$$2.2EI\theta_B + 0.6EI\theta_C - 0.375EI\delta - 99.17 = 0 \quad \text{-------- > (7)}$$

$$\text{and } M_{CB} + M_{DC} = 0 \quad (4) + (5)$$

$$112.5 + 1.2EI\theta_C + 0.6EI\theta_B + EI\theta_C - 0.375EI\delta = 0$$

$$112.5 + 2.2EI\theta_C + 0.6EI\theta_B - 0.375EI\delta = 0 \quad \text{-------- > (8)}$$

$$\text{also } M_{AB} + M_{BA} + M_{CB} + M_{DC} + 80 = 0$$

$$-13.33 + 0.5EI\theta_B - 0.375EI\delta + 13.33 + EI\theta_B - 0.375EI\theta\delta + EI\theta_C - 0.375EI\delta$$

$$+ 0.5EI\theta_C - 0.375EI\delta + 80 = 0$$

$$1.5EI\theta_B + 1.5EI\theta_C - 1.5EI\delta + 80 = 0 \quad \text{--------- > (9)}$$

By solving (7), (8) and (9) we get

$$EI\theta_B = 72.65$$

$$EI\theta_C = -59.64$$

$$EI\delta = +66.34$$

Final moments:

$$M_{AB} = -13.33 + 0.5(72.65) - 0.375(66.34) = -1.88 \quad \text{KNM}$$

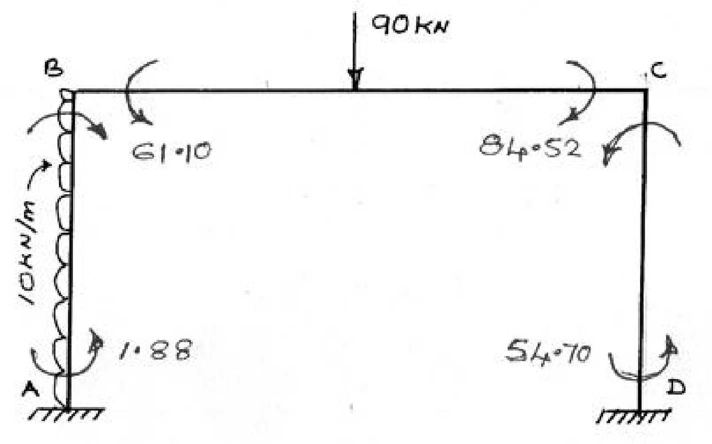
$$M_{BA} = +72.65 - 0.375(66.34) = 61.10 \quad \text{KNM}$$

$$M_{BC} = -112.5 + 1.2(72.65) = +0.6(-59.64) = -61.10 \quad \text{KNM}$$

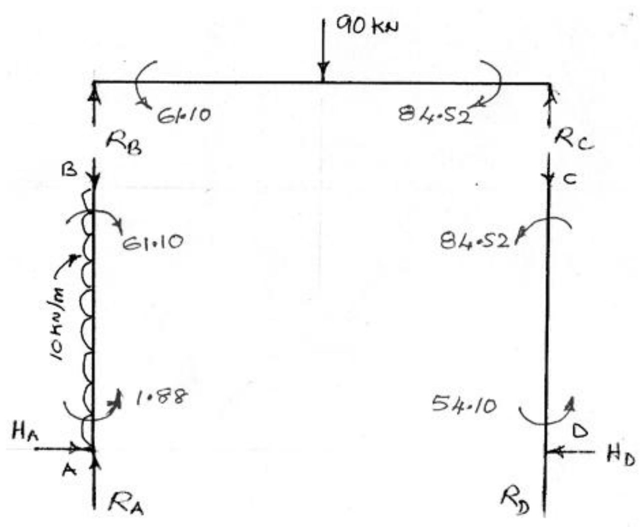
$$M_{CB} = 112.5 + 1.2(-59.64) + 0.6(72.65) = 84.52 \quad \text{KNM}$$

$$M_{CD} = -59.64 - 0.375(66.34) = -84.52 \quad \text{KNM}$$

$$M_{DC} = 0.5(-59.64) - 0.375(66.34) = -54.70 \quad \text{KNM}$$



Reactions: Consider the free body diagrams of various members



Member AB:

$$H_A = \frac{61.10 - 1.88 - 10 \times 4 \times 2}{4}$$

$$= -5.195 \text{ KN} \qquad \text{-ve sign indicates direction of } H_A \text{ is from right to left}$$

Member BC:

$$R_C = \frac{84.52 - 61.10 + 90 \times 5}{10} = 47.34 \text{ KN}$$

$$\therefore R_B = 90 - R_C = 38.34 \text{ KN}$$

Member CD:

$$H_D = \frac{84.54 + 54.7}{4} = 34.81KN$$

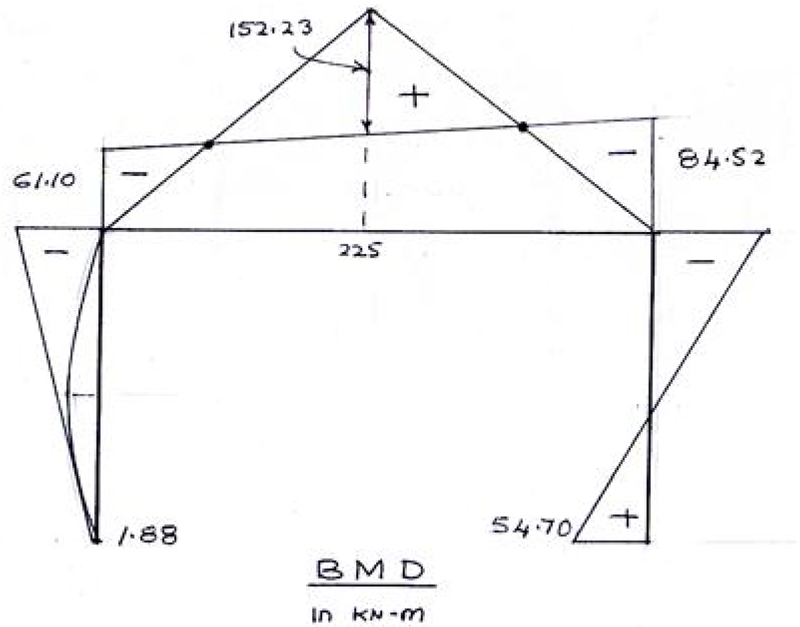
Check

$\Sigma H = 0$

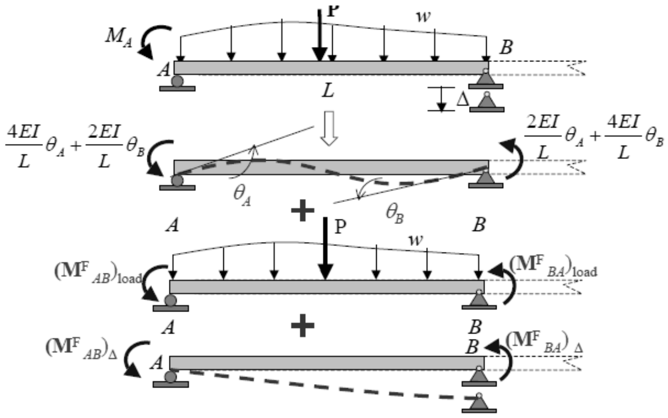
$H_A + H_D + 10 \times 4 = 0$

$-5.20 - 34.81 + 40 = 0$

Hence okay



4.7 SIMPLIFICATION FOR HINGED END



$$M_{AB} = M_A = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B + (M_{AB}^F)_{load} + (M_{AB}^F)_\Delta \quad ---(1)$$

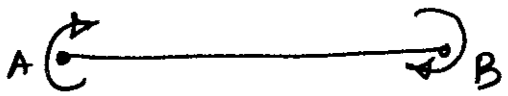
$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B + (M_{BA}^F)_{load} + (M_{BA}^F)_\Delta \quad ---(2)$$

$$Eliminate\ \theta_A\ by\quad \frac{2(2)-(1)}{2}:\quad M_{BA} = \frac{3EI}{L}\theta_B + [(M_{BA}^F)_{load} - \frac{1}{2}(M_{AB}^F)_{load}] + \frac{1}{2}(M_{BA}^F)_\Delta + \frac{M_A}{2}$$

QUESTION BANK

PART-A

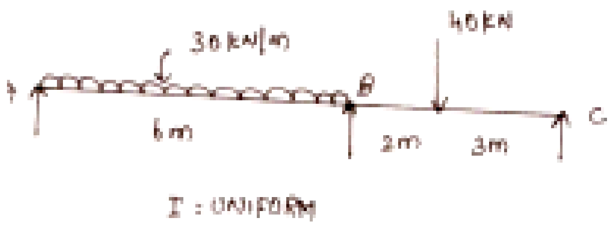
1. State relative merit of moment distribution method over slope deflection method.
2. Name the three classical force methods used in the analysis of continuous beams.
3. What are the limitations of slope deflection method?
4. Draw the deflected shape of the beam shown below



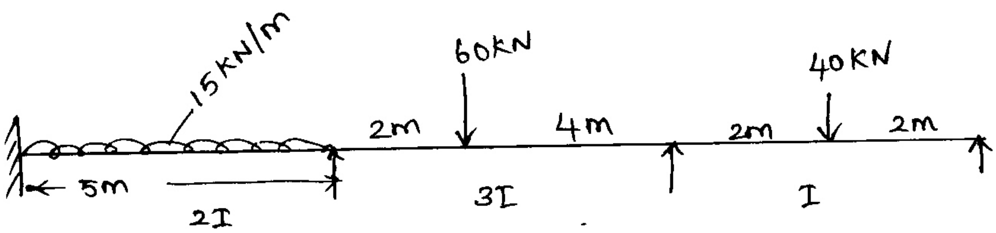
5. Write down the equilibrium equations used in slope deflection methods.
6. Why is slope deflection equation method known as stiffness method?
7. What are the basic assumptions made in slope deflection method?

PART-B

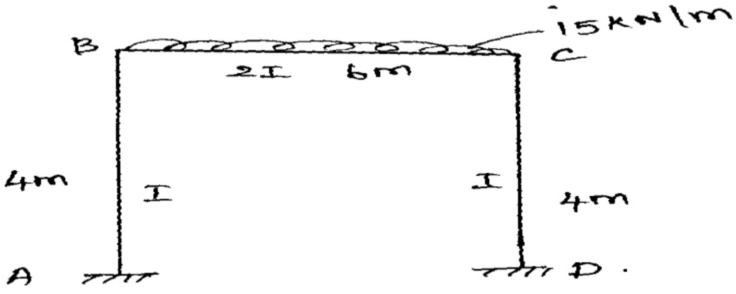
1. A continuous beam is built in at A and is supported over rollers at B and C, as shown in figure. AB=BC=12m. The beam carries a uniformly distributed load of 30kN/m over AB and a point load of 240kN at a distance of 4m from B on span BC. B has a settlement of 30mm. $E=2*10^5 N/mm^2$; $I=2*10^9 mm^4$. Analyze the beam by slope deflection method.



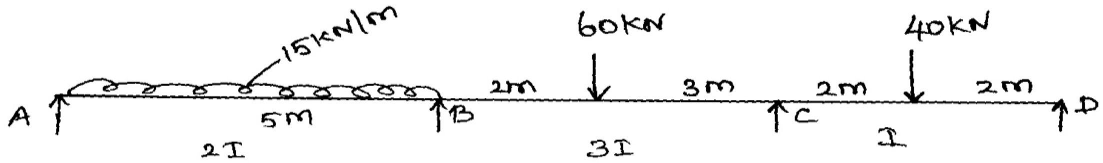
2. ABC is a continuous beam with constant EI throughout its length. The end supports A and C are fixed and the beam is continuous over middle support B. Span BC is uniformly loaded with 10 kN per meter length while a concentrated vertical downward load of 100kN acts at the midspan of AB. Calculate the moments by slope deflection method.
3. Analyse the continuous beam given in figure by slope deflection method and draw the B.M.D



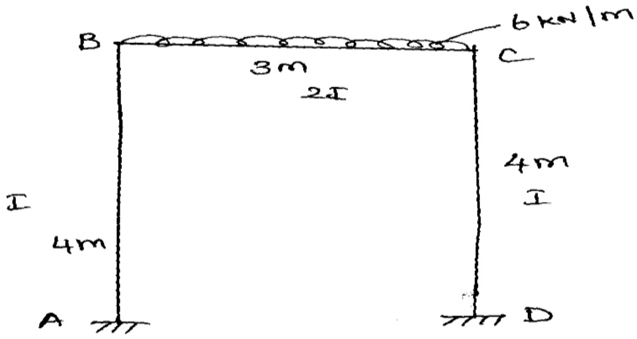
4. Analyse the frame given in figure by slope deflection method and draw the B.M.D



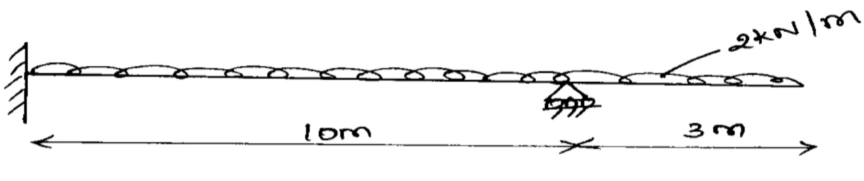
5. Analyse the continuous beam given in figure by slope deflection method and draw the B.M.D



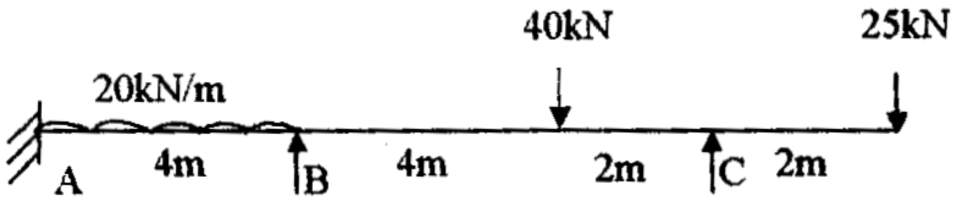
6. Analyse the frame given in figure by slope deflection method and draw the B.M.D.



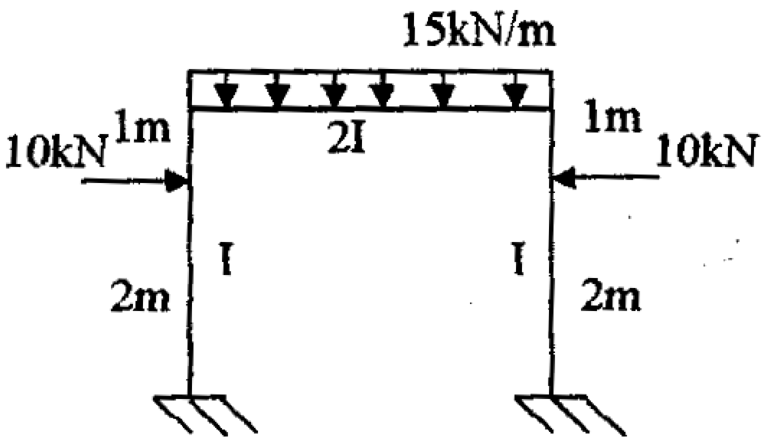
7. Analyse the continuous beam given in figure by slope deflection method and draw the B.M.D



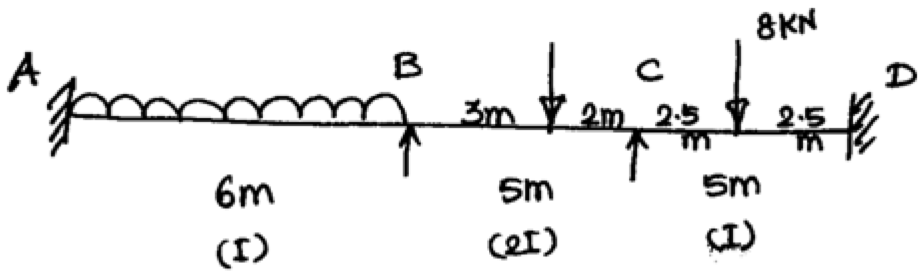
8. Using slope deflection method, determine slope at B and C for the beam shown in figure below. EI is constant. Draw free body diagram of BC.



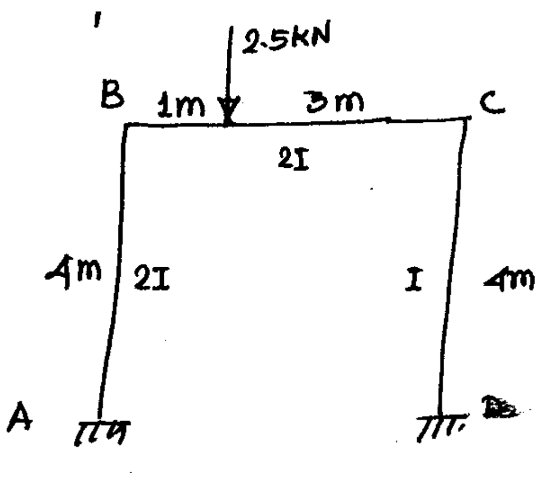
9. Analysis the frame shown in below by the slope deflection method and draw the bending moment diagram. Use slope deflection method.



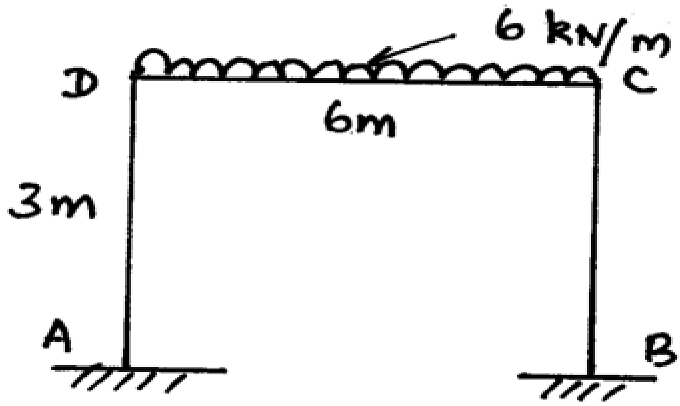
10. A continuous beam ABCD consist of three span and loaded as shown in figure end A and D are fixed using slope deflection method Determine the bending moments at the supports and plot the bending moment diagram.



11. A portal frame ABCD, fixed at ends A and D carriers a point load 2.5kN as shown in figure. Analyze the portal by slope deflection method and draw the BMD.



12. Using slope deflection method analyse the portal frame loaded as shown in figure. EI is constant.



13. Using slope deflection method analyse a continuous beam ABC loaded as shown in figure. The ends A and C are hinged supports and B is a continuous support. The beam has constant flexural rigidity for both the span AB and BC.